fulfills the first condition and the second states that tangency conditions are applied to the $\varphi_i^{(2)}$ problem. Thus, $r\varphi_{i_r}^{(1)} = A(x)$ must vanish, since it vanishes at the body surface. It follows, on matching potentials, that $\varphi_i^{(1)} = B(x)$ equals $\varphi_o^{(1)}(x,0)$. On the body $r = \tau R(x)$ surface pressures can be evaluated using $\epsilon^2 \varphi_{i_r}^2 = \tau^2 R'^2$ and the result that $\varphi_{i_x}^{(1)}(\epsilon x, r) = \varphi_{o_x}^{(1)}(x,0)$. To evaluate the right side of the foregoing equation we need the solution for $\varphi_i^{(2)}$. The choice $\delta = \epsilon^{1/2}$ or $\delta = \epsilon$ leads to Laplace's equation with the solution $\varphi_i^{(2)} = C(x)\log r + D(x)$ which satisfies tangency conditions. Evaluation on the body surface leads to $\lim_{r\to 0} \epsilon\delta r\varphi_{i_r}^{(2)} = \epsilon\delta C(x) = \tau^2 RR'$, and so,

$$\lim_{\rho\to 0} \epsilon\rho \frac{\partial\varphi_o^{(1)}(x,\rho)}{\partial\rho} = \lim_{r\to\infty} \epsilon\delta r \frac{\partial\varphi_i^{(2)}}{\partial r} = \tau^2 RR'$$

since $\epsilon\delta r\varphi_{i_r}^{(2)}$ is independent of r. Hence $\epsilon = \tau^2$ as opposed to $\epsilon \sim \tau^{2/3}$ in the near-planar two-dimensional case.

The stretching $\delta = \epsilon^{3/2}$ is the one of interest and leads to the following crossflow equation for $\varphi_i(\epsilon x, r)$,

$$\left(1 - \frac{6}{5} M_\infty^2 \varphi_{i_r}^2\right)\varphi_{i_{rr}} + \left(1 - \frac{1}{5} M_\infty^2 \varphi_{i_r}^2\right)\frac{\varphi_{i_r}}{r} = 0$$

with the solution

$$r\varphi_{i_r}\left|1 - \frac{1}{5} M_\infty^2 \varphi_{i_r}^2\right|^{5/2} = \mathcal{C}(x)$$

The correct branch is the one that identifies with classical theory for large r. For small r's the compressible correction removes the usual logarithmic singularity in φ_i. The effect of this nonlinearity, important near the body, must be transferred to the outer flow. This is accomplished by first determining $\mathcal{C}(x)$ by evaluating the preceding equation on the body surface, that is,

$$\mathcal{C}(x) \cong \frac{\tau^2}{\epsilon} RR' \left[1 - \frac{M_\infty^2 \tau^2 R'^2}{5\epsilon^2}\right]^{5/2}$$

Next the same function is evaluated for the outer flow, giving

$$\rho\varphi_{o_\rho}^{(1)}\left[1 - \frac{M_\infty^2 \delta^2}{5}\varphi_{o_\rho}^{(1)2}\right] \cong \rho\varphi_{o_\rho}^{(1)} + O(A^3)$$

Choosing $\epsilon = \tau^2$ produces the boundary condition for the outer flow,

$$\lim_{\rho\to 0} \rho\varphi_{o_\rho}^{(1)}(x,\rho) \cong RR'\left(1 - \frac{M_\infty^2 R'^2}{5A}\right)^{5/2}$$

where $M_\infty^2 R'^2/5A < 1$. The outer flow therefore sees an effective body slope reduced from its classical value and which, for small enough A's, decreases to zero. As before it is possible to show that $\varphi_i^{(1)} = \varphi_o^{(1)}(x,0)$.

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Driver Gas Contamination in a High-Enthalpy Reflected Shock Tunnel

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IT is well known that the utility of reflected shock tunnels is seriously limited by premature driver gas contamination of the test gas.[1] Davies and Wilson[2] have developed an explanation of this effect, which is consistent with measurements made in relatively low-enthalpy shock tunnels, where the primary shock Mach number was less than 6 (Refs. 2 and 3). However, their theory indicates that early contamination should not occur if the shock tunnel is operated at primary shock Mach numbers which are less than a value near the tailored interface level. In experiments with a high-enthalpy shock tunnel,[4] it has been found that early contamination persisted at shock Mach numbers down to 60% of the tailored interface value. In this Note, their theory is extended to take account of these results.

Davies and Wilson base their explanation of early contamination on the bifurcation which occurs at the foot of the reflected shock wave as it interacts with the wall boundary layer in the shock tube. As shown in Fig. 1a, if the bifurcation persists as the reflected shock is transmitted through the driver gas, then the gas which passes through the bifurcation region is decelerated through two oblique shocks, thereby suffering a smaller change in velocity than the gas which passes through the normal reflected shock. Thus, the gas which passes through the bifurcation region has a velocity towards the contact surface, which causes it to penetrate the contact surface as a jet along the walls, and thereby, to contaminate the test gas. Now, bifurcation persists if the minimum stagnation pressure P_{st} in the boundary layer is such that some of the boundary-layer fluid cannot negotiate the shock pressure rise. Davies and Wilson calculated P_{st} from the Rayleigh supersonic pitot formula, i.e.,

$$P_{st}/P_3 = [(\gamma+1)M_b^2/2]^{\gamma/(\gamma-1)}[2\gamma M_b^2/(\gamma+1) - (\gamma-1)/(\gamma+1)]^{1/(1-\gamma)}$$

where P_3 is the pressure ahead of the transmitted shock, γ is the ratio of specific heats of the boundary-layer gas, and M_b is the minimum Mach number in the boundary layer with respect to the transmitted shock. They assumed that the boundary layer was composed of test gas, and M_b was the value at the wall. As already noted, this yielded results which correlated satisfactorily with experiments at low enthalpies.

The high-enthalpy experiments[4] were performed with helium driver gas, at primary shock Mach numbers ranging from 13 to 23. Piezoelectric transducers were used to measure the speed of the transmitted shock system u_T after the contact surface had come to rest following completion of the shock reflection process. Estimates had indicated that bifurcation was more probable at this speed than at any earlier stage in formation of the transmitted shock. The pressure ratio across this transmitted shock system P_T/P_3 was obtained by combining measurements of the pressure after shock reflection with the contact surface pressure, as calculated

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from M_s, the measured primary shock speed. Using these measurements, the theory of Davies and Wilson predicted that shock bifurcation and early contamination would not occur. This was in conflict with the experimental evidence, which is described subsequently. It is worth mentioning that an alternative criterion for contamination due to Markstein[5] was similarly unsatisfactory, in that it also indicated that early contamination would not occur.

Now, Byron and Rott[6] have pointed out that, at high primary shock speeds, M_b does not occur at the wall. In fact, for a perfect gas, with a Prandtl number of unity, it can be shown that

$$M_b^2 = 2\{[u_T/a_I M_s - 1]^2/[1+2M_s^2/(\gamma-1)]-1\}$$

where a_I is the initial speed of sound in the test gas. When values of M_b calculated from this relation were used with the theory of Davies and Wilson, it was found that, for $\gamma=1.4$, $P_{st}<P_T$ when $M_s \geq 16$, indicating that bifurcation and early contamination could be expected for shock speeds above this value. This conclusion was in qualitative agreement with the experimental results which showed that the time delay to onset of contamination increased strongly with decreasing M_s at about $M_s = 16$.

Having established that transmitted shock bifurcation was possible, the theory of Davies and Wilson could be used to estimate the velocity of the driver gas wall jet. Referring to Fig. 1a, it was assumed that the pressure P_f at the foot of the bifurcated shock wave was given by $P_f = 0.55\ P_T$. Then the Mach number of the flow approaching the bifurcated shock M_T was obtained by noting that, since the transmitted shock system brings the driver gas to rest, it was possible to write the momentum relation

$$P_T/P_3 - 1 = \gamma_D M_3^2 (1+u_T/u_3)$$

where γ_D is the ratio of specific heats of the driver gas, M_3 is the driver gas Mach number, and u_3 is the contact surface velocity. This allowed M_3 and $M_T = M_3(1+u_T/u_3)$ to be derived from the measurements of P_T/P_3 and u_T, together with a contact surface velocity u_3, which was deduced from measurements of M_s. Combining P_f and M_T with the velocity u_3+u_T of the flow approaching the bifurcated shock, then yielded the velocity immediately downstream of the leading shock A of the bifurcation. The entropy rise across the trailing shock B was negligible, and so the final velocity of the wall jet u_j could be obtained by assuming an isentropic compression to the pressure P_T. Transforming to laboratory coordinates, the wall jet velocity then became $u_j - u_T$.

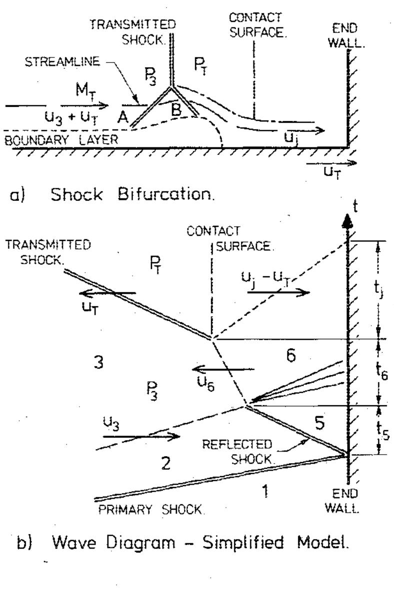

Fig. 1 Illustration of bifurcation of the reflected shock transmitted through contact surface, and a time-distance diagram of the shock-contact surface interaction.

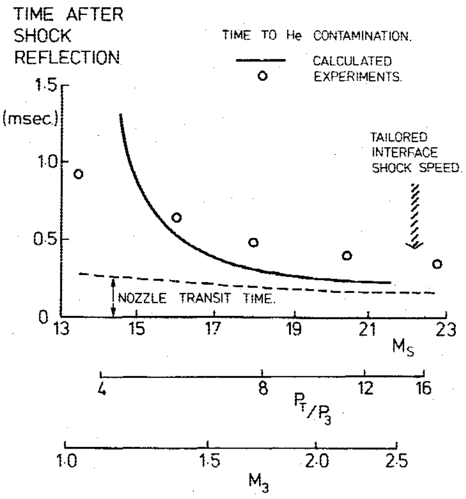

Fig. 2 Comparison of theoretical and experimental results for onset of driver gas contamination in the test section flow.

In using this velocity to estimate the time to contamination of the test section flow, the simplified model of Fig. 1b, appropriate to undertailored operation, was used. It was assumed that after the reflected shock arrived at the contact surface, the contact surface velocity u_6 remained constant until the surface reached its equilibrium position, at which it came to rest. At this point, the bifurcated transmitted shock originated, leading to the wall jet, which moved with velocity $u_j - u_T$ through the quiescent test gas. Thus, the total time delay, after shock reflection, for the driver gas to reach the end wall of the shock tube was calculated as $(t_5+t_6+t_j)$, as displayed in Fig. 1b. By adding this time to the nozzle transit time, the time for the test section flow to become contaminated was obtained.

Results of these calculations are compared with experiment in Fig. 2. For the calculations, the distance between the primary shock and the contact surface following it was obtained by using Mirels' analysis.[7] For interest, the nozzle transit time is shown, as well as scales for P_T/P_3 and M_3. The experimental results, which are described in detail in Ref. 4, were obtained with a mass spectrometer located in the test section. They show the time at which the helium molar concentration equalled 0.1.

The theoretical curve shows a steep rise in the time to contamination at $M_s = 15$, while the measurements show that this rise occurs more gradually. This is consistent with results obtained by previous authors at lower shock speeds (e.g., Figs. 3 and 6, Ref. 2), and may be regarded as a reflection of the many simplifying assumptions in the analysis. These are necessary to yield a theoretical result. The results indicate that, with attention to the choice of the criterion for boundary-layer separation, the driver gas contamination mechanism proposed by Davies and Wilson can be used satisfactorily to explain the early onset of shock tunnel driver gas contamination at high enthalpies.

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An Explanation of the Turbulent Round-Jet/Plane-Jet Anomaly

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Nomenclature

$C_\mu, C_{\epsilon 1}, C_{\epsilon 2}, C_{\epsilon 3}$	= turbulence model constants
℄	= centerline value
D/Dt	= $(\partial/\partial t) + U_i(\partial/\partial x_i)$, rate of change along a mean streamline
k	= kinetic energy of turbulence
P	= production rate of k
r	= radius in axisymmetric coordinates
s_{ij}	= rate-of-strain tensor
U_i	= mean-velocity vector
$\overline{u_i u_j}$	= Reynolds-stress tensor
V	= radial velocity (axisymmetric flow)
x_i	= position vector
$y_{1/2}$	= jet half-width
δ_{ij}	= Kronecker delta
ϵ	= rate of dissipation of k
$\mu_{\rm eff}$	= effective viscosity
$\sigma_k, \sigma_\epsilon$	= turbulence model constants
χ	= vortex-stretching invariant
ω_{ij}	= rotation tensor

Introduction

THE use of turbulence models to calculate the properties of free turbulent flows is now quite common.[1-3] Models that solve transport equations for two or more quantities have the potential advantage of generality since they require no direct empirical input such as a mixing-length specification. Their only empirical input is five or six constants which, for generality, are supposed to take the same values in all flows. However, this generality is found not to exist. Using the values of the constants appropriate to boundary-layer flows, the velocity field in a two-dimensional plane jet is calculated quite accurately, but large errors occur for axisymmetric jets. Specifically, the spreading rate of the round jet is overestimated by about 40%. Experimental data indicate that the round jet spreads about 15% less rapidly than the plane jet, while its calculated spreading rate is 15% greater.

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The discrepancy between calculated and measured values is found with both mean-flow and Reynolds-stress closures. Here, the simpler mean-flow closure is considered, and attention is focused on the k-ϵ model. For constant (unit) density flows, this model determines the Reynolds stresses through the isotropic viscosity hypothesis,

$$\overline{u_i u_j} = \frac{2}{3} k\delta_{ij} - \mu_{\rm eff}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{1}$$

where the effective viscosity is given by

$$\mu_{\rm eff} \equiv C_\mu k^2/\epsilon \tag{2}$$

k, the kinetic energy of turbulence ($\frac{1}{2}\overline{u_i u_i}$), and ϵ, the rate of dissipation of k, are determined from transport equations:

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_i}\frac{\mu_{\rm eff}}{\sigma_k}\frac{\partial k}{\partial x_i} + P - \epsilon \tag{3}$$

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_i}\frac{\mu_{\rm eff}}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_i} + \frac{\epsilon}{k}(C_{\epsilon 1}P - C_{\epsilon 2}\epsilon) \tag{4}$$

P is the rate of production of kinetic energy:

$$P = -\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} \tag{5}$$

Commonly used values for the constants C_μ, $C_{\epsilon 1}$, $C_{\epsilon 2}$, σ_k and σ_ϵ are 0.09, 1.45, 1.90, 1.0, and 1.3, respectively. With these values, wall boundary layers and the plane jet are well represented.

Previous Work

In order to obtain accurate calculations of round jets, some modification to the model is required. Changing $C_{\epsilon 1}$ to 1.6 produces the desired effect but, in so doing, any notion of generality has to be abandoned. Three attempts have been made previously to introduce a modification while retaining some semblance of generality. All of these make reference to centerline values (℄) and involve modifying either $C_{\epsilon 1}$ or $C_{\epsilon 2}$:

Launder et al.[1]

$$C_{\epsilon 2} = 1.92 - .0667\left\{\frac{y_{1/2}}{2U_{℄}}\left(\left|\frac{dU_{℄}}{dx}\right| - \frac{dU_{℄}}{dx}\right)\right\}^{0.2} \tag{6}$$

McGuirk and Rodi[4]

$$C_{\epsilon 1} = 1.14 - 5.31\frac{y_{1/2}}{U_{℄}}\frac{dU_{℄}}{dx} \tag{7}$$

Morse[5]

$$C_{\epsilon 1} = 1.4 - 3.4\left(\frac{k}{\epsilon}\frac{\partial U}{\partial x}\right)^3_{℄} \tag{8}$$

$y_{1/2}$ is the distance from the centerline to the location where the velocity is half the centerline velocity. (Launder et al. also made a modification to C_μ but this has no great effect for jets.) The different modifications to $C_{\epsilon 1}$ and $C_{\epsilon 2}$ produce the same desired effect for self-similar jets but differ slightly in the developing region; here, Morse's proposal fares best.

No convincing physical explanation is provided to justify these modifications and, in addition, there are two separate objections to them. These objections, though separate, both stem from the use of centerline values. First, the modifications imply "action at a distance"; a change in the centerline conditions is supposed immediately to affect the extremities of the jet. This is physically implausible. Second,